Names:

Group Quiz MA-141-001 Fall 2019

*This problem will count as a quiz grade. You may work on this in groups of up to 3 students. Turn in only 1 solution per group. Be sure to include all group members' names. Have fun!*

## Background

Transgenic (i.e., genetically modified) mice are a critical tool for scientific research. The production of genetically modified embryos, however, is a very delicate procedure. It involves the injection of modified DNA, dissolved in fluid, into the pronucleus of a fertilized oocyte (i.e., egg cell), which is then transferred into the oviducts of a surrogate mother mouse. During the microinjection procedure, however, the egg cell is susceptible to rupture if fluid is injected into the cell too quickly. In other words, excessive strain on the cell membrane due to rapidly increasing internal volume can cause the cell to burst. Specifically, if the surface area of the approximately-spherical cell increases more quickly than the membrane's lipid bilayer structure can allow, rupture will occur. In this problem, we will investigate the relationship between the surface area and volume of a growing spherical cell to determine what choice of fluid injection rate will avoid cell rupture.

Figure 1: Microinjection of transgenic DNA into fertilized murine oocyte.

## Problem Statement

A murine oocyte (i.e., mouse egg cell) has an average radius of $30\mu m$. We shall approximate the cell to be spherical. Recall that the formulas for the surface area and volume of a sphere are as follows:

$$A = 4\pi r^2 \qquad V = \frac{4}{3}\pi r^3$$

Suppose that the cell will rupture if at any point the surface area of its membrane increases at a rate greater than $25\frac{\mu m^2}{s}$.

(a) Differentiate both sides of the Volume formula with respect to time. Solve for $\frac{dr}{dt}$.

(b) Suppose the default setting on the microneedle/pipette apparatus injects fluid at a rate of $500\frac{\mu m^3}{s}$. Assuming the cell has the average radius of $30\mu m$, at what rate does the radius of the cell increase when the fluid is injected?

(c) Differentiate both sides of the Surface Area formula with respect to time. Using the results from ($b$), determine the rate at which the surface area of the cell increases when injected. Does the cell burst?

(d) Determine the maximal rate at which fluid can be injected into the cell without causing rupture.

(e) Ovine (i.e., sheep) oocytes are slightly larger, with a radius of $50\mu m$. Repeat part ($d$) for the ovine oocyte. Does the ovine cell burst under the default injection rate of of $500\frac{\mu m^3}{s}$?